HEAT EXCHANGE AND HYDRAULIC RESISTANCE DURING LAMINAR CONDENSATION OF NITROGEN TETROXIDE IN A VERTICAL PIPE

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A method of calculating the heat exchange and hydraulic resistance during film condensation in a vertical pipe is discussed. The results of the calculation are compared with experimental data on the condensation of nitrogen tetroxide and water.

The calculation of heat-exchange apparatus with condensation in bundles of pipes operating in parallel and with a variable cooling intensity along the rows and along the length of the pipes is possible only with the use of methods permitting the computation of the local and average values of the coefficients of heat exchange and hydraulic resistance. The conditions of condensation of turbulent and laminar streams of vapor with a laminar film of condensate are widespread in practice. However, the known calculating recommendations do not possess enough universality, mainly because of the serious assumptions simplifying the solution in the calculation of the velocity of the condensate film and the shear stresses at the phase interface.

Comparable solutions of the problem of laminar condensation of a rapidly moving vapor at a vertical plate are obtained in [1-3] with the force of gravity neglected. An equation determining the average coefficient of heat exchange ($\bar{\alpha}$) for a vertical pipe was derived analytically in [4], in which the inertial forces and the influence of the transverse flow of vapor (V''_r) were ignored while the shear stress at the phase interface (τ_{if}) was defined by analogy with the case of a dry smooth wall. Using the Blasius resistance law for the conditions of constant vapor velocity in a vertical pipe an expression was obtained in [5] for calculating $\overline{Nu}$. The functions from [1-5] are valid at high vapor velocities V'' and do not provide for the limiting transition to the conditions of heat exchange when the vapor is motionless. The approximate equation of [6, 7] has definite advantages in this respect. Satisfactory generalization of experimental data on the condensation of water vapor is achieved by these functions in the ranges of the parameters with the dominant influence of the factor being taken into account or by the introduction of empirically chosen constants. In this connection, the experimental data on the condensation of nitrogen tetroxide, which is distinguished by a high density of the vapor and by low values of the viscosity of the phases and of the surface tension, cannot be generalized satisfactorily by the known calculating functions.

In the general case the motion of a condensate film is determined by the forces of gravity and friction and by the effect of momentum transfer by the condensing vapor. The main difficulties consist in the calculation of the coefficient of friction at the phase interface (c_f) in the wave mode of film flow with allowance for the wave height and the change in the vapor velocity profile.

We obtained a calculating function for c_f on the basis of the equation [8] for adiabatic annular flow, refined in [9]:

$$c_f = c''_f\left[1 + 300\left(\frac{\delta}{D} - \frac{5}{\mathrm{Re}''}\sqrt{\frac{2}{c''_f}}\right)\right]. \quad (1)$$

In (1) the thickness of the viscous sublayer of the gas stream (the last term) is subtracted from the actual wave height, which is proportional to δ. The quantity c''_f is calculated from the equation for turbulent flow in a smooth dry pipe. It is obvious that (1) is valid when $\delta/D > 5/\mathrm{Re}''\sqrt{2/c''_f}$, while in the opposite case $c_f = c''_f$. Under these conditions of descending, comoving, annular, two-phase flow, in the absence of separation and of annular waves of large height, the constancy of the relative wave amplitude ($\lambda = 0.46$ [10]), close to the value of

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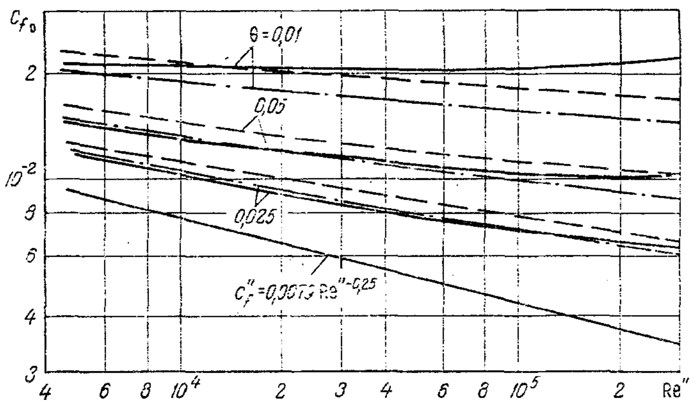

Fig. 1. Dependence of the coefficient of friction for a dry smooth wall with uniform suction: solid curve) calculation according to [15]; dashed curve) by the equation $c_{f_0} = c''_f\,(1 + 17.5\,\mathrm{Re}''^{0.25}\cdot\theta)$ [13]; dash-dot line) by (2).

$\lambda = 0.48$ calculated analytically in [11], is usually retained. In an ascending stream with constancy of the liquid flow rate, λ increases (from 0.48 to 0.86 [10]) with a decrease in velocity, so that Eq. (1) evidently corresponds better to descending comoving flow.

With condensation under the action of a transverse flow the velocity profile of the vapor stream changes, with an increase in the velocity gradient at the phase interface and a decrease in the thickness of the viscous sublayer, and therefore these factors must be allowed for in (1).

The coefficient of friction for turbulent flow with mass exchange (c_{f_0}) can be calculated by various methods [12-15]. For the problem under consideration it is most reasonable to use the results of [15], which is a development of the calculation method worked out in [13] and [14]. Simple solutions in closed form for c_{f_0} and the longitudinal velocity profile are obtained in [15] using the Stevenson boundary law [16].

An equation for the calculation of c_{f_0} is recommended in [13] which gives overstated results in the requisite region of $\theta = V''_y/V''_x < 0.08$ and $\mathrm{Re}'' < 10^5$ in comparison with a calculation by the method of [15] (Fig. 1). Better agreement with the calculation is achieved by replacing the constant 17.5 by 14. Then

$$c_{f_0} = c''_f(1 + 14\mathrm{Re}''^{0.25}\theta). \tag{2}$$

It must be noted that, according to [13-15], c_{f_0} in (2) does not allow for the variation of the momentum of the vapor stream along the channel.

An increase in θ with a fixed V''_x, which is equivalent to an increase in the heat load during condensation, leads to an increase not only in c_{f_0}, but also in the total frictional resistance because of a sharp increase in the "profile" component. The latter is determined by the height of the waves "projecting" into the turbulent core of the stream, proportional to the thickness of the condensate film and the size δ_V of the viscous vapor stream. Profiles of the longitudinal vapor velocities calculated by the method of [15] for different values of θ are shown in Fig. 2 (solid lines). In the presence of surface mass exchange the variables u^+ and y^+ do not allow one to obtain a universal velocity profile, and the curves $u^+ = f(y^+)$ are layered at different θ. Allowance for the decrease in δ_V and the increase in the effective wave height in Eq. (1) can be made by multiplying the last term by the ratio of the thicknesses of the viscous layers with mass exchange and without it ($\theta = 0$). A function for the calculation of this correction was obtained on the basis of an approximation of the calculated data. It was arbitrarily assumed that $\delta_{V_0}/\delta_V = y^+_0/y^+$; the values of y^+_0 and y^+ were found from the coordinates of the points of intersection of the curves of the turbulent profile and $u^+ = y^+$. In reality, these curves have a flat conjunction (see Fig. 2), and in the transition from dimensionless to absolute values of V_x and y one should allow for τ_w, but allowance for these factors adds unjustified complication to the calculation. As a first approximation, the correction can be calculated by the equation

$$K_\theta = \frac{\delta_{V_0}}{\delta_V} = \mathrm{Re}''^{-0.01}\exp[-(50\theta)^{1.6}]. \tag{3}$$

In the range of θ = 0.025–0.01 and Re'' $\leq 10^5$ the departures of (3) from the calculation are less than 4.5%.

The characteristics of a wavy surface depend to a large degree on the value of the surface tension [17-19]. In the expression of [19] for calculating the coefficient of friction in the presence of condensation its influence is allowed for by the factor $\sigma^{-2/3}$, and in [17] in the form of the ratio σ/σ_{H_2O}. On the basis of an analysis of the available data we adopted the correction

$$K_\sigma = \left(\frac{\sigma}{\sigma_{H_2O}}\right)^{-\frac{2}{3}}. \tag{4}$$

With allowance for the foregoing, the expression for calculating the coefficient of interphase friction in the laminar condensation of a turbulent vapor stream has the following form:

$$c_{f_{if}} = c_{f_0}\left[1 + 300K_\sigma\left(\frac{\delta}{D-2\delta} - \frac{5}{\mathrm{Re}''}\sqrt{\frac{2}{c_{f_0}}}K_\theta\right)\right]. \tag{5}$$

In (5), c_{f_0}, K_θ, and K_σ are calculated by (2), (3), and (4), respectively.

Under these conditions the heat exchange is determined by the thermal resistance of the condensate film, since it is assumed that

$$\frac{dT}{dy} = \frac{\Delta T}{\delta} \quad (\Delta T = T_{if} - T_w), \quad \alpha = \frac{\lambda}{\delta}.$$

The velocity of the film surface (V_{x_σ}) and the average velocity over the cross section ($\bar{V}_\delta$) can be calculated by equations obtained through double integration of the equation of film motion,

$$\mu\frac{d^2V_x}{dy^2} + g(\rho - \rho'') = 0,$$

with the standard assumptions for such a problem (P, T = const, etc. [20]). The conditions of the interaction of the phases at the interface are

$$T_{if} = T'' = T_s, \quad -\lambda\left(\frac{\partial T}{\partial y}\right)_{if} = rV''_y\rho'', \quad P_{if} = P'',$$

$$\pm\mu\left(\frac{\partial V_x}{\partial y}\right)_{if} = V''_y\rho(V''_x - V_{0_1}) + 0.5c_{f_{if}}\rho''(V''_x - V_{0_2})^2,$$

and those at the wall are

$$V_y = V_x = 0, \quad -\lambda\left(\frac{\partial T}{\partial y}\right)_w = \alpha(T'' - T_w).$$

The shear stresses τ_{if} are determined by the transfer of the momentum of the vapor, which depends on the difference between the velocities of the vapor and the film surface (V_{0_1}), and by the frictional forces, proportional to the difference between the velocity of the vapor and the phase velocity of the waves (V_{0_2}) when "profile" losses have the dominant influence. According to [8, 10, 11, 18], one can take $V_{0_2} \approx 2.5\bar{V}_x$.

As a result of integration, we obtain

$$V_{x_\delta} = -\frac{\delta}{\mu}\left[g(\rho - \rho'')\delta \pm V''_y\rho''V_{0_1} \pm \frac{1}{2}c_{f_{if}}\rho''V_{0_2}^2\right], \tag{6}$$

$$\bar{V}_x = \frac{\sigma}{\mu}\left[\frac{1}{3}g(\rho - \rho'')\sigma \pm \frac{1}{2}\left(V''_y\rho''V_{0_1} \pm \frac{1}{2}c_{f_{if}}\rho''V_{0_2}^2\right)\right]. \tag{7}$$

The equations for the film thickness at a distance x from the entrance can be obtained by solving (7) jointly with the equation

$$dG = \frac{q}{r}dx = d(\rho\bar{V}_x\delta) = \frac{\lambda}{\delta r}\Delta T\Delta x. \tag{8}$$

For the conditions of ΔT = const we have

$$\frac{\mu\lambda\Delta Tx}{\rho r'} = \frac{g}{4}(\rho - \rho'')\delta^4 \pm \frac{1}{3}V''_y\rho''V_{0_1}\delta^3 \pm c_{f_{if}}\rho''V_{0_2}^2\delta^3, \tag{9}$$

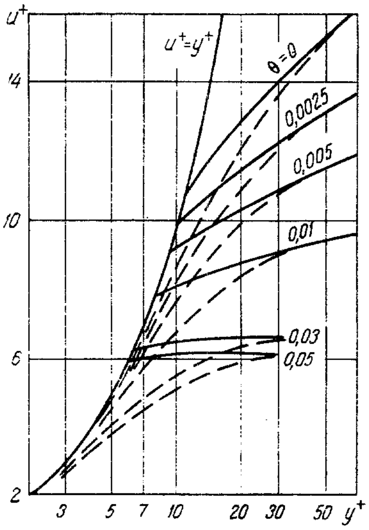

Fig. 2

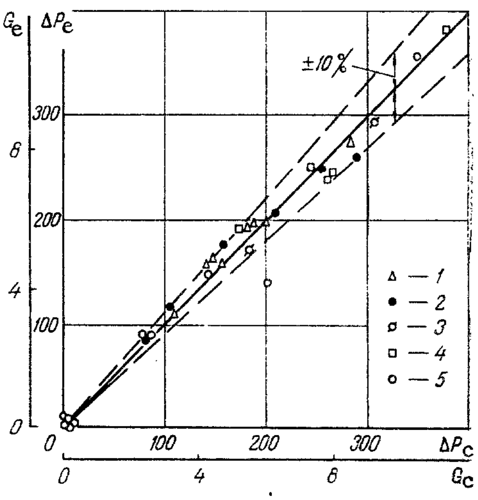

Fig. 3

Fig. 2. Velocity profiles of a turbulent stream with suction in the boundary region at Re" = 13,500.

Fig. 3. Comparison of calculated and experimental data on the condensate flow rate and the static pressure drop; total condensation: 1) H_2O [24]; 2) N_2O_4, 7.5 bar; 3) N_2O_4, 4 bar; 4) partial condensation of N_2O_4, 7.5 bar; 5) ΔP, N_2O_4. G_e, G_c, kg/h; ΔP_e, ΔP_c, kg/m^2.

while for q = const we have

$$\frac{\mu q x}{\rho r} = \frac{1}{3} g(\rho - \rho'')\delta^3 \pm \frac{1}{2}(V''_y V_{0_1} \pm c_{f_{if}} V^2_{0_2})\delta^2\rho''. \tag{10}$$

The variation in δ along the length of a section $\Delta x = x_{n+1} - x_n$ is calculated by (9) and (10) with δ^m replaced by $(\delta^m_{n+1} - \delta^m_n)$. The + or − signs in (6), (7), (9), and (10) are chosen as a function of the relative directions of the velocities of the film surface and the vapor and of the gravity vector.

Wave formation lowers the thermal resistance of a condensate film, which is usually taken into account by the correction factor ε_υ. In the condensation of a stationary vapor, $\bar{\varepsilon}_\upsilon = 0.835\mathrm{Re}^{-0.11}$, according to [21]. Motion of the vapor intensifies the process of heat and mass transfer in the film; moreover, the local values of ε_{υ_x} must somewhat exceed $\bar{\varepsilon}_\upsilon$. Therefore, with allowance for the results of an analysis of experimental data for the conditions of Re" > 25,000, we took $\varepsilon_{\upsilon_x} = 0.9\mathrm{Re}^{-0.11}$.

The momentum of a laminar film is relatively low, so that it is admissible to calculate the change ΔP in static pressure along the length of a pipe by confining the analysis to the vapor stream. The calculating equation for determining ΔP in a sufficiently small segment Δx of pipe length is obtained from the equations of continuity and motion:

$$r\frac{\partial V''_x}{\partial x} + \frac{\partial}{\partial r}(rV''_y) = 0, \tag{11}$$

$$V''_x \frac{\partial V''_x}{\partial x} + V_y \frac{\partial V''_x}{\partial r} = g - \frac{g dP}{\rho dx} + \nu \frac{\partial}{r \partial r}\left(r\frac{\partial V''}{\partial r}\right). \tag{12}$$

Solving (11) and (12) jointly, and neglecting the third term on the right side of (12) because of its smallness or equality to zero when q = const, we have

$$-\Delta P = \beta\frac{\rho}{g}(\bar{V}''^2_{x_n} - \bar{V}''^2_{x_{n+1}}) - \rho\Delta x \pm c_{f_{if}}\frac{\rho V^2_{0_2}\Delta x}{g(R-\delta_n)} \pm \frac{2\rho V_{0_1} V''_y \Delta x}{g(R-\delta_n)}, \tag{13}$$

where β is a parameter of the velocity profile allowing for the nonuniformity of the velocity distribution over a cross section of the pipe,

$$\beta = \frac{\overline{V_x'''^2}}{\overline{V_x''}^2} = \frac{\int\limits_0^1 \left(\frac{V_x''}{V_{\max}''} \right) r dr}{2 \left[\int\limits_0^1 \left(\frac{V_x''}{V_{\max}''} \right) \bar{r} d\bar{r} \right]^2}, \qquad (14)$$

and has a complicated dependence on Re" and θ [22]; at moderate values of Re" and θ the quantity β is close to unity.

Experimental data on the condensation of N_2O_4 of stable composition [23] and of water vapor [24] in vertical pipes were analyzed using the method under consideration. The experiments on N_2O_4 condensation in a pipe with a length of 781 mm and an inner diameter of 8 mm were conducted at P = 7.5 bar (with total and partial condensation) and 4 bar (with total condensation). The vapor velocity at the entrance was 0.71-7.10 m/sec, the Reynolds number of the film at the end of the pipe was 123-559, and the thermal load was 11-170 kW/m². The water vapor was condensed (without evaporation) in pipes with a length of 1500 mm and diameters of 10.1 and 19 mm at pressures of 7.5 and 28 bar, vapor velocities at the entrance of 0.22-7.22 m/sec, thermal loads of 15-121 kW/m², and film Reynolds numbers of 131-447. Unfortunately, the original experimental data on the pressure drop during condensation under these conditions are absent from the literature, and therefore ΔP was calculated only for the conditions of the experiments with N_2O_4. For the analysis we chose experiments with parameters corresponding to the conditions of the given problem and with a minimal error. In connection with the absence of recommendations on the calculation of Re_{cr} in the condensation of moving vapor we arbitrarily took 400 as the most likely value of Re_{cr}.

The calculation was carried out on a Minsk-22 computer by the method of successive approximations in steps whose size was increased from 0.5 to 55 mm over a length of 285.5 mm and then remained constant. We assumed that ΔT is constant over the length of a step. In the initial section with a smooth film (Re $\leq$ 6) the coefficient of friction was determined by (2). With laminar flow of the vapor c_{fif} = 16/Re". As a result of the calculation we determined the local values of the parameters of the vapor stream and the condensate film and the overall data on the condensate flow rate and the static pressure drop. Since the condensate flow rate in the experiments (G_e) was determined with a minimal error in comparison with α, a comparison of the experimental and calculated data on the condensate flow rate as well as of the data on ΔP is given in Fig. 3.

A comparison of the results of calculations of the length of condensation (required for the condensation of G_e under the experimental conditions) made by the present method (L_c) and by the Nusselt theory (L_{Nu}) showed the following. Despite the similar values of ρ'' and V_x'' under the conditions of the experiments with N_2O_4 and H_2O, the degree of action of the vapor stream on the condensate film differed sharply. For example, with $V_{in}'' \leq 3.9$ m/sec for H_2O with total condensation $L_{Nu}/L_c \leq 1.05$, while for N_2O_4 with the same velocity $L_{Nu}/L_c \leq 1.269$ and 1.752 with total and partial condensation, respectively. A similar divergence of the data is also observed at other velocities.

Attempts to correlate the experimental data on H_2O and N_2O_4 using various dimensionless complexes did not yield positive results. In particular, with equality of the parameter [25]

$$\frac{\rho''}{\rho'} \mathrm{Fr} \left(\frac{\mathrm{Ga}}{\mathrm{Re}} \right)^{1/3}$$

the divergence in the values of L_{Nu}/L_c for H_2O and N_2O_4 increased two or more times. The use of the complex [25] (ρ''/ρ')Fr somewhat improves the results, but not to the required extent. This indicates the inadequate universality of the available criterial functions and the reason for the unsatisfactory generalization of the experimental data on N_2O_4 by functions obtained for H_2O.

The comparative data presented in Fig. 3 confirm the possibility of using the method under consideration for calculating the process of laminar condensation in a pipe.

NOTATION

δ, mean thickness of condensate film, m; x, y, longitudinal and transverse (radial) coordinates; D, pipe diameter, m; $\theta = V_r/V_x$, ratio of suction velocity at the wall to axial velocity; $u^+ = V_x/\sqrt{\tau_w/\rho}$, dimensionless axial component; $y^+ = y/\sqrt{\tau_c\rho/\nu}$, dimensionless transverse coordinate; σ_{H_2O}, surface tension of water under standard conditions; $\bar{r} = r/R$, dimensionless radius; Fr, Froude number; Ga, Galileo number. Indices: ", vapor stream, 0, conditions with suction; v, viscous boundary layer of stream; w, wall; if, phase interface.

LITERATURE CITED

1. D. A. Kazenin, Inzh.-Fiz. Zh., No. 11 (1964).
2. I. G. Shekriladze, Soobshch. Akad. Nauk GruzSSR, 35, No. 3 (1964).
3. I. (J.) G. Shekriladze and V. I. (J.) Gomelauri, Intern. J. Heat Mass Transfer, 9, 581 (1966).
4. H. Hartmann, Chem.-Ing.-Tech., 33, No. 5, 343 (1961).
5. M. Jakob, S. Enk, and H. Eck, Phys. Z., 36, No. 3 (1935).
6. L. D. Berman and Yu. A. Tumanov, Énergomashinostroenie, No. 5 (1964).
7. L. D. Berman, Tr. Tsentr. Nauchno-Issled. Proektno-Konstr. Kotloturbinnyi Inst., No. 101, 262 (1970).
8. G. B. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill, New York (1969).
9. R. M. Neddermann and C. Shearer, J. Chem. Eng. Sci., 19, No. 6, 423 (1964).
10. B. I. Kokobeev, V. A. Malyusov, and N. M. Zhavoronkov, Khim. Mashinostr., No. 7, 31 (475) (1961).
11. P. L. Kapitsa, Zh. Eksp. Teor. Fiz., 18, No. 1, 48 (1948); P. L. Kapitsa and S. P. Kapitsa, Zh. Eksp. Teor. Fiz., 19, No. 2, 105 (1949).
12. A. I. Leont'ev, Teploénergetika, No. 9 (1972).
13. R. B. Kinney and E. M. Sparrow, Trans. ASME, 92, No. 1, 117 (1970).
14. L. Merkine, A. Solan, and Y. Winograd, Trans. ASME, 93, No. 2, 242 (1971).
15. T. Y. Na, Trans. ASME, 94, No. 3, 700 (1972).
16. T. N. Stevenson, J. Fluid Mech., 33, Part 4, 715 (1968).
17. A. P. Colburn, in: Proceedings of the General Discussion on Heat Transfer, September 11-13, 1951, Institute of Mechanical Engineers, London (1951), p. 448.
18. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Akad. Nauk SSSR, Moscow (1952).
19. V. V. Nashchokin and A. V. Klevtsov, in: The Investigation of Transfer Phenomena in Complex Systems [in Russian], Nauka i Tekhnika, Minsk (1974), p. 124.
20. S. S. Kutateladze, Principles of the Theory of Heat Transfer [in Russian], Nauka, Novosibirsk (1970).
21. N. V. Zozulya, in: Heat Transfer and Thermal Modeling [in Russian], Akad. Nauk SSSR (1959), p. 278.
22. J. K. Aggarwal, M. A. Hollingsworth, and G. R. Mayhew, Intern. J. Heat Mass Transfer, 15, 1585 (1972).
23. T. M. Batishcheva, V. T. Derov, and L. I. Kolykhan, Izv. Akad. Nauk BelorusSSR, Ser. Fiz. Energ. Nauk, No. 3 (1975).
24. Yu. T. Illarionov, Author's Abstract of Candidate's Dissertation, Leningrad (1974).
25. V. P. Isachenko and F. G. Salomzoda, Teploénergetika, No. 5 (1968).